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Various Optimization Applications Using Mathematical Models - A Review

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Abstract: *There are different applications, which can be optimized using Mathematical models. In this paper, we review some papers related to those optimization applications. Mathematical models used in optimizing those applications, are also discussed. Two different types of optimization techniques used in those applications are also presented.*

Key Words: *Application, Optimization, Scheduling, Networking , Assignment, Path Discovery.*

1. INTRODUCTION:

Operation Research is a managerial application of higher Mathematics. In Operation Research analytical methods are used to solve problems and decisions were taken of different management problems. In Operations research, real life problems are broken into some basic components. A basic operations research problem mainly broken down into following parts: (1) identifying a problem, (2) deriving a Mathematical model of the problem with variables (3) finding solution of the derived model (4) testing the solutions to analyse the success and (5) implementing the solution obtained in actual real-world problem. Hence Mathematical analysis method is used to solve any Operations research problem.

Optimization is very significant word in the field of Operations Research. In **[27]** Optimization models, which is also called as mathematical programs in Operations Research, is defined as a problem with decision variables. These problems seek values to maximize or minimize objective functions of the decision variables and this process subject to certain specific constraints on variable values, which expresses the limits on possible decision choices. Hence, to do optimization of any real time problems, Mathematical models are derived and solution is obtained. Generally, in most of the Mathematical problem, there are certain fixed restrictions and few available resources. According to **[1]** optimization is an optimal decision that is done with considering restrictions and by using available resources.

In **[23]** Ammar E. E. and Emsimir A. A was derived a Mathematical optimization model for solving integer linear programming problems. In this study basic concepts of some rough intervals are reviewed. They also presented a methodology for optimizing fully rough integer linear programming problems. During this optimization process they found various rough value optimal solutions and decision rough integer variables, in which all decision variables and parameters in the constraints and the objective functions are rough intervals. In other words, we can define the word optimization as to take appropriate action that gives us the best and most effective result by using various available resources.

In operations research there are various types of optimizations techniques. Optimization problem formulation, methods of optimization and solution techniques are presented by Ms J. Renuka Kumari **[1]** in her paper. Population based methods are also explained in this paper.

In **[26]** Kanika Tyagi and Kirti Tyagi done a comparative study on various optimization techniques. In this paper, they were discussed about artificial bee colony optimization algorithm, particle swarm intelligence optimization, ant colony optimization algorithm and genetic algorithm optimization techniques. Evaluation of all this optimization techniques were discussed here. With this methodology, performance and application wise comparison was done in this paper.

In **[29]** another review of optimization techniques was done by three Mathematicians, Sanket A. Pandya, Vijay V. Mehta and Nirav K. Meghapara. Out of many other optimization techniques, they considered four important techniques such as, Genetic Algorithm, Ant Colony Method, Honeybee Algorithm and Simulated Annealing Algorithm. Each and every method was analysed briefly, and, in the conclusion, they stated that in modern developed optimization techniques are used to solve non-differentiable as well as non-Linear optimization problems. Generally, these problems cannot be explained by conventional optimization techniques. There are many other authors who have discussed about various optimization techniques. Many real time applications are optimized using these techniques.

In **[30]** a similar review paper on optimization techniques was written by G. Venter.
In this paper the author discussed about local optimization algorithms. Newton's algorithm is one of an unconstrained algorithm. This algorithm was derived from a second-order Taylor series expansion of the objective function with considering x^0 as an initial design point. With Newton's method, two other methods called as Fletcher-Reeves and the Broyden Fletcher-Goldfarb-Shanno (BFGS) methods for solving unconstraint optimization problems was explained in this paper.

Linear programming is one of the techniques to optimize any Mathematical real-life model. In **[3, 11, 18,19]** linear programming models were developed to solve different real-life problems. But the result of linear programming is not always coming in integer value. Sometimes while solving a linear programming problem, we can get fractional solutions. But in some real-life problem fractional solutions are not possible. So, because of that in many situations integer linear programming is used to optimize a problem. In **[2, 4, 5, 6, 9, 12, 16, 17]** integer linear programming models were derived where only integral solutions are considered. Binary integer linear programming **[6, 13]** is another type special of integer linear programming model. In this model, all the decision variables x_j, where j=1, 2…n can only take a value of 0 or 1 and they are called as e binary variables. All the coefficients of objective function are > 0 (i.e., non-negative) . All the decision variables are ordered according to their coefficients in objective function with satisfying the condition $0 \leq c_1 \leq c_2 \leq \ldots\ldots \leq c_n$.

Similarly, another Mathematical and computational optimization technique is known as Genetic algorithm (GAN). Genetic algorithms **[7, 8, 14, 15]** use the similar Mathematical principles to optimize various complex real-life systems. The basic principle behind GAs is to minimize a linear objective function. We can also rely on this type of algorithm to satisfy linear inequality constraints.

2. PRELIMIARIES:

As we know there are many optimization techniques. Out of all techniques, Integer linear programming (ILP) and Genetic algorithm (GAs) of Operations research can be used in many real-life applications areas. These types of optimizations are used to optimize (either maximize or minimize) any real-life process or a Mathematical model **[23]** to get the optimum desired result. In **[32, 33]** applied integer programming modelling technique and methods of integer linear programming is discussed in brief and the solution procedure of ILP model was explained.

This integer linear programming applications can be solved by using various methods such as brunch and bound method **[2,13, 17,23]**, cutting plane method **[2]**, by using duality theory of linear programming **[3]**. In **[4]** integer linear programming models was formulated for the Stochastic Ground-Holding Problem, and it was solved by using two algorithms known as RBS algorithm and Compression Algorithm. In case of some binary integer linear programming model **[6]** integral constraints and 0-1 Constraints were derived with many other real time constraints to get the optimal results. In **[13]** binary integer programming model was developed by considering Graph theory concept and Greedy Algorithms notation and limits. In **[9]** telecommunication networks were optimized by developing an integer programming. The model was developed with the concept of 4-color-graph-problem and in the paper **[9]** they linked cell phone networks with Mathematical graphs.

Another type of optimization technique is genetic algorithm **[7, 8, 14, 15, 47, 48, 49,50]**. Many real time applications can be solved using this algorithm. In **[7]** genetic algorithm was derived with different genetic operators such as Crossover operator and Mutation operator. In **[8]** a genetic algorithm (GA) based solution procedure was suggested for the mathematical model which provides cell configuration. In **[14]** a genetic algorithm was developed, and the hard constraints were tested to ensure that all the optimization solutions obtained were valid. In **[15]** a genetic algorithm was designed based on the characteristics of the problem. The experimental results verify the feasibility and effectiveness of the genetic algorithm proposed. In **[47]** an improved Genetic Algorithm to solve the scheduling problem of college English courses was proposed. Here a GA model was developed for a college to optimize the courses running for the subject English. In **[48]** a genetic algorithm was developed to optimize the Investment Strategy of an organization. In **[49]**. a genetic algorithm (GA) which approaches to the portfolio design based on market movements and asset valuations. A similar timetabling model was developed in **[50].** In this study genetic algorithm is used to solve the Timetable Problem.

Sometimes various software's are also used to find out the solutions of different optimization techniques. In **[8, 13, 39]** LINGO 13 software was used to find solutions of genetic algorithm model as well as integer linear programming model. By running premium solver for Excel software, we can solve any integer linear programming model **[12]** as well as a general linear programming model **[18].** In **[16]** at the end of the paper the integer linear programming model was solved using Gurobi software and its python interface.

Now let us study some of these real-life applications of optimization in brief.

2.1 PROJECT INVESTMENT OPTIMIZATION:

Every company uses their funds in various projects to earn maximum profit. But this investment should be done in such a way that, the profit of the company is maximized. So, they must set a project investment policy for the company. These project investment policies of a company can be optimized using different Mathematical models.

In **[17]** Zhao, C. published a paper on application of integer programming in project investment. In this study the basic expression of integer programming model was derived as

Objective function: max(min) z $=\sum_{j=1}^{n} c_j x_j$such that

Constraints: $\sum_{j=1}^{n} a_{ij} \leq (=,\geq) b_j$ (i = 1, 2, ---, m),

Non-negativity constraints: $x_j \geq 0, with\ x_j$is an integer (j = 1, 2, ---, n), and

To solve this derived mathematical model branch and bound method of integer linear programming was used.

In **[18**] three Mathematician Mustafa, A. O., Sayegh, M. A. H., and Rasheed, S. was done research on the topic called as: application of linear programming for optimal investments in software company. All software projects of the company are divided into four categories: (1) Mobile Applications. (2) Web Applications. (3) Desktop Applications and (4) Enterprise Resource Planning (ERP). Different problem categories were analysed using constraints. Those constraints are as follows: the Number of Expert Programmers, The Number of Testers, Storage Size, Fixed Expenses Per Project etc. A linear programming model is formulated on these constraints and model was solved using MS excel tool to obtain the optimal results.

In **[19]** another study was done by Nordin, S. Z., Johar, F., & Abu, N. on the topic Linear Programming Model for Investment Problem in Maximizing the Total Return. They developed a model with objective function as maximize $f\ (x_i)\ =\ \sum_{i=1}^{n} r_i x_i$ (which calculates total return of the company).

And the one of the constraints as $\sum_{i=1}^{n} x_i = A\ \forall i \in N,$

where A is the total amount received, N is the set of FD, r_i is the rate of return and x_i is the amount of money invested.

2.2 SCHEDULING AND AIR TRAFFIC FLOW MANAGEMENT:

Scheduling is one of the important applications of integer linear programming. Some of the examples of scheduling problems are Processor scheduling, Bandwidth scheduling, Airport gate scheduling, Repair crew scheduling. In processor scheduling, various jobs are executed on a CPU in a multitasking operating system. In this scheduling all the users submit jobs to web servers, and they receive results after some time. Users can also submit batch computing jobs to a parallel processor. In bandwidth scheduling all the users call other persons and this scheduling need bandwidth for some period. In Airport gate scheduling airlines require gates for their flights at an airport. Similarly, Air traffic management is also another special type of scheduling problem. All these problems can be optimized using different optimization techniques of operations research.

In **[2]** initially Ryan, D. M., & Foster, B. A has explained the set partitioning model of Mathematical programming and graph theory.

The model derived by them is as follows:

SPP (Set Partitioning Problem) :

Minimized Z= c^T x , x ∈ $\mathbb{R}^{n}$ subject to

Ax = e, where e^T = (1 , 1, 1-------------, 1) ∈ $\mathbb{R}^{m}$

and $x_j = 0\ or\ 1$, j = 1, 2, ----------, m

Where A is an $m \times\ n$ matrix of zeros and ones and c is the objective coefficient vector. In the model each of the columns a_j , j= 1, 2, - - - - - - n , of A represents a duty or schedule with an associated cost c_j .The corresponding variable xj can be considered as the probability that the jth column is included in the solution. Where the jth duty aj has the elements a $a_{ij} = 1$ if duty performs the task i

= 0, otherwise. They took vehicle scheduling and bus crew scheduling problem as a case study in their paper. They considered various scheduling preferences in this study. Brunch and bound and cutting plane method of integer linear programming was used to solve the derived model.

In **[4]** Homan, R. L, done a study on Integer Programming models for ground holding in Air traffic flow management. In this paper a Mathematical model was formulated for the Stochastic Ground-Holding Problem. The ground holding problem (GH) model requires the following assumptions.

Assumption 1: (Discrete time horizon)
There is a fixed time horizon. That horizon has been discretized into T equally sized contiguous time periods, t =1 ;2; --------------T.

Assumption 2:
Number of flights will land is known advance and for each flight f the schedule time or arrival is denoted by a_j.

Assumption 3:
For each time span t , the arrival acceptance rate of the airport is b_t.
Let C_f be the cost of delaying flight f for one period and let α >1 be a fixed parameter.
We define for each f and each t, a binary variable, $X_{f\,t}$, such that
$X_{f\,t}$= 1; if flight f is assigned to time interval t
= 0; otherwise. Two algorithms, RBS algorithm and Compression Algorithm was used to solve the derived Mathematical model and optimal solution is obtained.

In **[5]** Sawik, T, was done on Integer Programming Approach to Production scheduling for Make-To-Order Manufacturing. He defined a model as: minimize number of tardy orders,
NTO= $\sum_{g\in G} u_g$, subject to the following conditions, order allocation constraints. $\sum_{t\in T, t\geq a_g} w_{gt} = o_g, g \in G$ and capacity constraints $\sum_{t\in T, t\geq a_g} w_{gt} = o_g, g \in \boldsymbol{G}$**.**
A model was developed on production scheduling for a single period order. At the end of the paper the models were represented in terms of computational approach. It was interpretated as follows: Any considered production system consists of the following m = 5 processing stages:
First are three flashing/flexing stages i = 1, 2, 3, where software required for the process is downloaded,
second state is postponement stage, where i = 4, and products for some orders are customized.
In the same way **last stage** is packing stage with i = 5, where products and required accessories are packed for shipping.

In **[39]**, Reza Ramezanian, Mohammad Saidi-Mehrabad and Donya Rahmani proposed a study on the topic mixed integer linear programming model for integrated scheduling and maintenance problem. In this paper they proposed a formulation of the MINLP model. LINDO 8 optimization solver is used to find the optimal solution.

In **[41]**, three researcher, F. Maaroufi Mars, H. Camus, O. Korbaa proposed a study on mixed integer linear programming model for scheduling the operating room. A MILP (mixed integer linear programming model) model is derived in this paper to minimize the make span of an operation theatre. To check the efficiency of MILP model they developed a CP (Constraint Programming) model on the same problem. With this a comparative study was done on this two-optimization method.

In **[45]** Wasakorn Laesanklang, Dario Landa-Silva and J. Arturo Castillo-Salazar proposed a mixed integer programming model with Decomposition to solve a related to workforce scheduling and routing problem. In this study a mixed integer programming model for real-life instances of a workforce scheduling and routing is derived. This model was derived by incorporating various constraint formulations and working region constraints.
are also added to the formulation of this model.

In **[46]** Can Celikbilek, Bulent Erenay and Gursel A. Suer presented a paper on Simulation and Mixed Integer Programming Optimization for Manufacturing and Transportation Scheduling.

2.3 CELLULAR NETWORKS:

A cellular network can be defined as a network of cells. This cell consists of different geographical regions. Connecting those regions using cell phones is known as cellular networking problem. Cellular networking problem is an application of graph theory. But linear programming models can be derived on this application. In **[9]** Borndörer, R., & Grötschel, M. gave lectures on the topic designing telecommunication networks by integer programming. In these two lectures they were covered the topic called as 4-color-graph-problem. In 1852 the Four Colour Problem was visualized by Francis Guthrie. While trying to color the map of counties of England Francis noticed that any map can be coloured using only four colors in such a way that adjacent regions get different colors. Borndörer and Grötschel linked cell phone networks with Mathematical graphs. They had developed a network optimization model for this telecommunication problem and used integer programming method to optimize the network model.

2.4 CASH FLOW MATCHING:

The system of cash flow matching is a systematic process in which a single entity or an organization or any individual company matches its cash outward flow (i.e., financial obligations) with its cash inward flow over a given stipulated time. This problem can be considered as a subset of immunization strategies in finance. To define the benefit of any pension plan, cash flow matching problem analysis is important. Cash flow matching problem is another important application of integer linear programming. Hence all Cash flow matching system can be formulated into a form of a Mathematical model.

In **[3]** three Mathematician Kocherlakota, R., Rosenbloom, E. S., and Shtu, E. S. W. was derived a model on cash flow which tell that the decision-maker may seek to find the investment portfolio $\{n_k\}$ by
minimizing total cost $\sum_k \mathrm{n_k}\, p_{\mathrm{k}}$, under the constraints
$\sum_k \mathrm{n_k}\, c_{\mathrm{k,t}} \geq \mathrm{l_t}$ for all t and $\mathrm{n_k} \geq 0$
where p_k denote the current price of one unit of kth security c, its cash flow at time t.
In general, the standard (primal) linear programming problem tends to determine a vector
x $\geq$ 0 in $\mathbb{R}^n$ which satisfies the system of m linear inequalities.
Ax $\leq$ b and maximizes the objective function given as

$$c^t x = c_1x_1 + c_2x_2 + ---------c_nx_n.$$

The derived model was solved by using duality theory of linear programming problem. In the duality theory given problem (primal) was converted into dual problem and after that solving process starts

2.5 MANUFACTURING CELLS OPTIMIZATION:

The manufacturing Cell Design Problem is also known as MCDP. This is a typical optimization problem in which we find application in lines of manufacture. The manufacturing cell problem consists in distributing machines in cells. This this distribution, the parts processed by each machine travel in the process of production in such a manner that productivity is greatly improved. Manufacturing cells optimization can also be done using optimization techniques. Mathematical models can be derived on this process to optimize the system.

In **[6]** Hassan, S. A., & Abed, S. Y. was developed a comprehensive integer programming. Model with special forms for optimal provision of multiple manufactures different constraints were set to model this problem. Those constraints are fixed cost coefficients, provision of the total demand, optimum number of production machines, needed area for importing/storing, stepped subcontracting, overtime, subcontracting stepping prices, Integral constraints and 0-1 Constraints.

In **[8]** Mathematician Shiyas, C. R., and Pillai, V. M. was done a study on the topic a zero-one integer programming model for the design of manufacturing cells. In this study a genetic algorithm (GA) based solution procedure was suggested for the mathematical model which provides cell configuration. In a complex search space, genetic algorithms are search techniques for global optimization. They constructed on the concept of the survival of the fittest among solution structures with structured, randomized search strategy in which new strong solutions displace weak solutions in successive iterations. Initially the variables are given a chromosome representation and after that initialization of population is done. The evaluation function and genetic operators are defined to construct the genetic algorithm.
The GA was solved using a software package known as LINGO.

2.6 CLASSROOM ASSIGNMENT AND TIMETABLING PROBLEM:

Classroom assignment and timetabling problem is a well-known real-life problem. A classroom assignment (or hotel room or interval scheduling) problem can be defined as to assign classes in such a way that it satisfies different time intervals of lecture in a day, to a room with one-to-one assignment. In this problem, two classes may not meet simultaneously in the same room(i.e., classroom changing is possible), nor may a class meet in two different rooms. Whereas any timetabling problems can be considered as specific types of scheduling problems which deal with assigning different subjects and teachers to the given time intervals. The assignment of subjects, teachers and courses are subject to certain hard constraints . These hard constraints should be achieved to get a feasible timetable. Similarly, there are some soft constraints which must be satisfied during forming a feasible schedule. A feasible schedule will optimize the full problem to get the best possible solution. In most of the educational institution's optimization is needed for this problem.

In **[7]** three researchers Aldy Gunawan, K. M. Ng and H. L. Ong was developed a Genetic Algorithm for the Teacher Assignment Problem for a University in Indonesia. In this paper, each chromosome is identified as a one-dimensional matrix. In phase I, the elements in the array represent teachers who teach these courses. In Phase 2. the order shown represents the different courses and course sections, and the elements in the array represent teachers who teach the course sections. By selecting of proper genetic representation, the problem in this paper was treated as a constraint free problem. Here every chromosome generated is a feasible solution. A mathematical programming model for this problem is formulated, which allows the possibility that courses could be taught by more than one teacher. In this study they used different genetic operators such as Crossover operator and Mutation operator.

In similar way another genetic algorithm was developed by researcher Achini Kumari Herath in **[14]** for university course timetabling problem. In this study the timetable was built around the following criteria: the rooms, professors, timeslots, courses/modules, and student groups. Hence a timetable class will encapsulate all these objects and will co-ordinate how different constraints interact with each other. At the end of the paper the hard constraints considered are as follows:

(1) lasses can only be scheduled in free classrooms.
(2) A professor can only teach one class at any one time.
(3) Classrooms must be big enough to accommodate the student group.

These constraints were tested using genetic algorithm approach to ensure that all the solutions obtained were valid.

Another genetic algorithm approach is found in **[15]** In this study three researcher Bei Wang, Yuejie Geng and Zhigang Zhangwere done a study on the topic applying genetic algorithm to university classroom arrangement problem. In this paper the classroom arrangement problem to be treated in such a way that it will reduce the moving distance generated by room changes between courses. Full time collection period was be divided into timeslots, and each timeslot consists of 90 minutes. Timeslot, course, class, and classroom are four entities considered in this problem. The class room arrangement problem is formulatd as :Problem { T , S , D , R , C},
where , T = { $t_1, t_2, - - - t_n$} is a set of time slots,
S = { $s_1, s_2, - - - s_m$} is a set of courses,
D = { $d_1, d_2, - - - - d_c$} is a set of classes.
Also, R = $r_1, r_2, - - - - r_s$} is a set of classrooms,
where $r_i = (n_i, l_i)$ and n_i, l_i denote the number and floor of the classroomm r_1 respectively,
C = { $c_1, c_2, - - - - c$} is a set of constraints.
In this paper, a model of the university classroom arrangement problem was established, and then the genetic algorithm was designed based on the characteristics of the problem. The experimental results verify the feasibility and effectiveness of the genetic algorithm proposed.

In **[11]** a group of Mathematicians, Sánchez-Partida, D., Martínez-Flores, J. L., and Olivares-Benítez, E. and Elias Olivares-Benítez was done a study on the topic an integer linear programming model for a university timetabling problem considering time windows and consecutive periods. In this study a linear programming model was developed based on the constraints and with the use of available resources. There are various subsets of decision variables as follows:
Isub_modo Sets of modalities. Face-to-face, video or mixed regardless of the school to which they belong.
Isub_cc1 and Isub_cc2 Sets of courses that require specific hours per week applied by the academic program coordinator.
Isub_cat Sets of courses taught by the professors, no matter the modality and the school to which they belong.
Jsub_req Sets of rooms classified by their properties as normal, special and laboratory belonging at buildings F and E where can be accommodated the courses.
Ksub_dia Sets of k periods for each day of the week in which can be scheduling the courses. **Ksub_vtc** Sets of k periods where professors cannot teach courses.
Weekly hours requested for each course.
Daily hours requested in consecutive periods for each course.
With these subsets we can reduce 175,968 decision variables. In this study the model is built on a set of decision variables defined below:
∀ i ∈ Isub_modo, Isub_cc1, Isub_cc2, Isub_cat; ∀ j ∈ Jsub_req; ∀ k ∈ Ksub_dia, Ksub_vtc
Where, I = Set of courses that belong to one modality and school,
J = Set of normal rooms, special rooms and laboratories belonging at building E and F {1...J}.

K = Set of total weekly periods {1...K}. The mathematical model derived with above consideration was validated in different instances and the model was solved with an optimization software called Lingo 13 unlimited version. After this an efficiency comparison was done between the results obtained by software and the administrative staff results.

In another study **[12]** Thongsanit, K., done a study on solving the course - classroom assignment Problem for a university. In this study the author was gathered information about all the courses of all department and developed a integer linear programming mathematical model. This linear programming (LP) model involves an optimization problem with linear objective functions and linear constraints. This derived LP model has three basic components.1) Objective of goal that is aimed to optimize the problem. 2) Constraints or restrictions that are needed to satisfy, for example a limited number of raw materials or labors. 3)Decision variables or the solutions, The non-negativity restrictions accounting for this requirement. The solution for the derived mathematical model is obtained using Excel' Premium Solver for Excel software. In this paper as an example of the course-classroom assignment problem on Monday was presented in the paper.

Another classroom assignment model was developed in **[16]** by three Mathematicians, Urban Rivero, L. E., Benıtez Escarcega, M. R., & Velasco, J. In this aper they developed a model for a case Study in Classroom Assignment problem. They were first set some basic definition of variables and after that they were defined a integer linear programming model on those variables.

They considered the following specific conditions for form their ILP model:

(1) University assigned the student load to the entire group without inference from the students.
(2) The university gives the professor-course schedule assignment in a previous stage.
(3) The courses have lengths between 1 hr and 4 hr with periods of 1/2 hr.
(4) The courses are taught from Monday to Friday from 7:00 a.m. to 10:00 p.m (30 periods of 1/2 hr).
(5) If a course has a schedule on a specific day, the same course cannot be assigned to a different time on the same day.
(6) The classrooms may have different capacities.
(7) Each course is associated with a group, and the capacity of the course is the capacity of the group. After designing the model, the author solved the model by running premium Solver for Excel software.

At the end of the paper the T the ILP model was solved using Gurobi software and its python interface and the result was obtained in few minutes. But at the last of this paper, they were mentioned that, in future, the classroom changes for a group during each day can be minimized further.

In **[28]** four researcher, Hayat Alghamdi, Tahani Alsubait , Hosam Alhakami, Abdullah Baz done a review of optimization algorithms for university timetable scheduling. In this paper they reviewed papers from the year 2015 to 2020 and classified all the papers according to the techniques and tools used for optimization process. In this study they have done a comparative study of all the techniques and listed down their advantages and disadvantages.

2.7 OPTIMIZATION OF LEARNING PATH DISCOVERY: -

In **[13]** a study was done on Linear Programming theory and Applications on the topic a Binary Integer Programming Model for Global Optimization of Learning Path Discovery. Path Discovery is basically a graphical problem which can be converted into a networking problem of operations research. In this paper three Mathematician, Belacel, N., Durand, G., & Laplante, F. was developed a Mathematical model with considering Graph theory concept and Greedy Algorithms notation and limits. The model was an binary integer programming model and was defined as:

Minimize

$\sum_{i=1}^{n}(\sum_{j=1}^{m}(Q_{i,j}+G_{i,j})x_i = \deg(X),$

subject to certain constraints

$Q_{i,j}\, x_i - \left(\sum_{k=1}^{i-1} G_{k,j}\, x_k\right) \times Q_{i,j} \leq 0$

for I = 2, 3, ---------, n-1 , for j = 1, 2 ,3, --------, m , $x_i \in \{0,1\}$,

= $\{x_i, i = 1, 2, 3, ------, n\}$ are the decision variables such that

$x_i = 1$ id the item I is selected.

= 0, otherwise

By using Branch-and-Bound (B&B) method the model was solved to get optimum result.

2.8 A FLEET ASSIGNENT MODEL: -

The word fleet management refers to all actions that is needed to take place to keep a vehicle fleet running on time, efficiently and within specified budget. It is a process that is used by a fleet manager. This fleet managers monitors the fleet activities and depending on that they make decisions about proper asset management, dispatch and routing, and vehicle acquisition and disposal. In this fleet management it is ensured that a fleet is meeting compliance requirements, continuously improving efficiencies, and reducing costs.

In **[10]**. Ozdemir, Y., Basligil, H., & Sarsenov, B. was done a case study on the topic a large-scale integer linear programming to the daily fleet assignment problem in Turkey. In this paper, they were discussing the fleet assignment model for one of the most important problems with which most of the airline companies must deal. This study provides an optimize model to reduce cost, and to optimize airlines fleets by using real case data. Hence in this paper a fleet Assignment Model was derived from available data. An Application in Turkish Airlines was solved using that model. The basic fleet assignment model (FAM), derived by them was as follows:

F = Set of flights,
K = Set of fleet types
C = Set of last nodes,
representing all nodes with aircraft grounded overnight at an airport in the network
M = Number of nodes in the network.

Index, i = Flight Index, j = Index for fleet, k = Index for nodes
Parameters, Ci,j = Cost of assigning fleet type j to flight I
N_j = Number of available aircraft in fleet type j
$S_{i,k}$ = +1 if flight i is an arrival at node k, -1 if flight i is a departure from node k
Decision Variables
$x_{i,j}$ = 1 if flight is assigned to fleet-type j,
= 0 otherwise
$G_{k,j}$ = integer decision variable representing number of aircraft of fleet-type on ground at node k.
The integer linear programming model is as follows:

$min \sum_{j\in K}\sum_{i\in F} C_{i,j}\, x_{i,j}$ subject to $\sum_{j\in K} x_{i,j} = 1\ , \forall i \in F$

$$G_{k-1,j} + \sum_{i\in F} S_{i,k}x_{i,j} = G_{k,j}\ \forall k \in M \ and\ \forall j \in K$$

$$\sum_{k\in C} G_{k,j} \le N_j\ , \forall j \in K$$

$$x_{i,j} \in \{\,0\,,1\} \ \forall\ i \in F\ and\ \forall j \in K$$

In the conclusion they stated that the solution to the mode derived by them generates
a minimum daily cost of fleet assignment of $732,872 and they optimized the number of aircraft that will be Grounded Overnight at each airport.

In **[44]** Arnita and Herman Mawengkang was done a study on mixed integer programming model. This mixed integer programming model; they applied on split delivery for vehicle routing problem with fleet and driver scheduling. In this paper the authors intended to develop an efficient mathematical model which optimizes one of the most economically importance problems which is known as optimizing logistic systems. Hence the main objective of this paper was to develop a MILP (mixed integer linear programming) model of split delivery vehicle routing.

3. CONLUSION:

From the above study we have analysed different Mathematical models. These Mathematical models are derived on real life problems and optimization techniques are applied on these models to get optimum result. In the same manner there are lot many recent real-life applications where Mathematical models can be derived, and optimization techniques can be applied to optimize the system. This can be considered as the future scope this review paper.

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